Performance Analysis of OFDM under DWT, DCT based Image Processing

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Abstract— **In this paper, the PSNR performance of conventional discrete cosine transform (DCT) - OFDM system is compared with discrete wavelet transform (DWT)-OFDM system. We use different error correcting codes viz; Reed Solomon, LDPC, DCT and DWT.**

Keywords- **DCT, DWT, OFDM, Reed Solomon, LDPC.**

I. INTRODUCTION

In Today's multimedia communication scenario, a growing demand emerges for high-speed, reliable, high-quality digital data. These trends place significant challenges to the parallel data transmission scheme which alleviates the problems encountered with serial systems. High spectral efficiency and resilience to interference caused by multi-path effects are the fundamentals to meet the requirements of today's wireless communication. The onset of Orthogonal Frequency Division Multiplexing (OFDM) has raised the wireless standards to 50 Mbps and higher, which creates a revolutionary in the wireless business.

The distribution of the data over many carriers means that selective fading will cause some bits to be received in error while others are received correctly. By using an error-correcting code, which adds extra bits at the transmitter, it is possible to correct many or all of the bits that were incorrectly received. The information carried by one of the degraded carriers is corrected, because other information, which is related to it by the error-correcting code, is transmitted in a different part of the multiplex.

OFDM systems

The basic idea underlying OFDM systems is the division of the available frequency spectrum into several sub carriers. To obtain a high spectral efficiency, the frequency responses of the sub carriers are overlapping and orthogonal, hence the name OFDM. This orthogonally can be completely maintained with a small price in a loss in SNR, even though the signal passes through a time dispersive fading channel, by introducing a cyclic prefix (CP). A block diagram of a base band OFDM system is shown in Figure 1. The binary information is first grouped, coded, and mapped according to the modulation in a "signal mapper". After the guard band is inserted, an N-point inverse discrete-time Fourier transform (IDFT) block transforms the data sequence into time domain. Following the IDFT block, a cyclic extension of time length, chosen to be larger than the expected delay spread, is inserted to avoid inter-symbol and inter-carrier interferences. At the receiver side, after passing through the analog-to-digital converter (ADC) and removing the CP, the DFT is used to transform the data back to frequency domain. Lastly, the binary information data is obtained back after the demodulation and channel decoding.
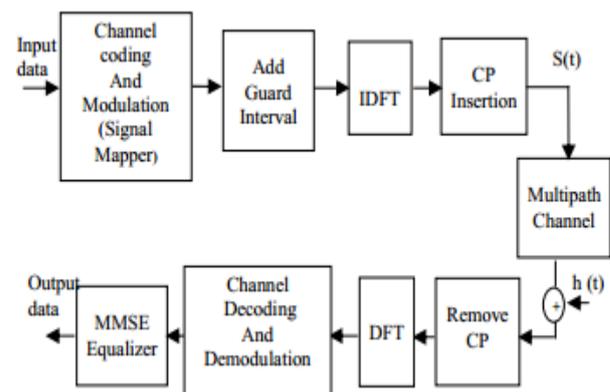

Fig.1.Base band OFDM system

Low-density parity-check

LDPC was first introduced by Gallager in his 1960 doctoral dissertation. After that in the year 1981 tanner introduced its graphical representation. In information theory, a low-density parity-check (LDPC) code is a linear error correcting code, a method of transmitting a message over a noisy transmission channel, and is constructed using a sparse bipartite graph.

In LDPC the message block is transformed into a code block by multiplying it with a transform matrix. Low density in the name implies low density of the transform matrix. That means number of 1s in the transform matrix is less.

A *(n, k)* LDPC encoder operates on an $m \times n$ sized *H* matrix where $m = n - k$. It is low density because the number of *1s* in each row w_r is « *m* and the number of *1s* in each column w_c is « *n*. A LDPC is regular if w_c is constant for every column and $w_r = w_c$ (n/m) is also constant for every row. Otherwise it is irregular. In LDPC encoding, the code word $(c_0, c_1, \ldots, c_0)$ consists of the message bits $(m_0, m_1, \ldots, m_k)$ and some parity check bits and the equations are derived from *H* matrix in order to generate parity check bits. The solution in solving the parity check equations can be written as:

$$Hc^T = 0$$

Where such mathematical manipulation can be performed with a generator matrix G. G is found from H with Gaussian elimination which can be written as follows:

$$H = [P^T : I]$$

And G is $G = [I : P]$

Hence, c code word is found for message word x as follows $c = xG = [x : xP]$.

Tanner introduced an effective graphical representation for LDPC. Not only provide these graphs a complete representation of the code, they also help to describe the decoding algorithm
Tanner graphs are bipartite graphs. The graphical representation for a typical (8, 4) LPDC encoding is shown in Fig. 1.
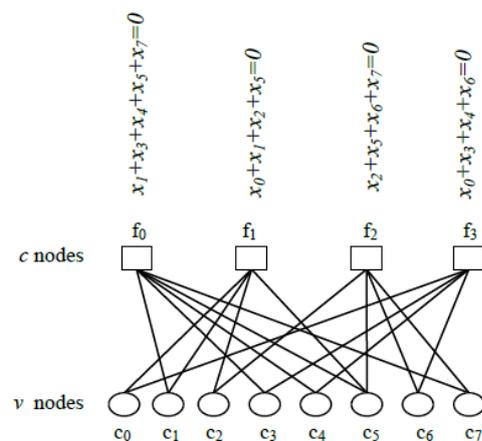

Fig.2. Graphical representation of a (8, 4) LDPC code.

The graphical representation utilizes variable nodes (v-nodes) and check nodes (c-nodes). The graph has four c-nodes and eight v-nodes. The check node f_i is connected to c_i if h_{ij} of H is a 1. This is important to understand the decoding. Decoding tries to solve the (n-k) parity check equations of the H matrix. There are several algorithms defined to date and the most common ones are message passing algorithm, belief propagation algorithm and sum–product algorithm. In this paper, we have employed sum-product decoding algorithm as presented in [1].

II. SYSTEM DESIGN

Reed Solomon Code
Reed–Solomon (RS) codes are non-binary cyclic error-correcting codes invented by Irving S. Reed and Gustave Solomon.
         LDPC is the best coding technique as far as the coding gain is concerned but encoder and decoder design is complex on the other hand the reed Solomon can achieve high coding rate and have low complexity.
Reed–Solomon termed as a systematic way of constructing codes that could notice and correct many random symbol errors. By adding t check symbols to the data, it can detect up to t erroneous symbols or we can say that it can correct up to ⌊t/2⌋ symbols. Besides, RS codes are appropriate as multiple-burst bit-error correcting codes, as an order of b + 1 consecutive bit errors can disturb at most two symbols of size b. The selection of t is depends on the designer of the code, and may be selected within wide limits.

For integers $1 \leq k < n$, a field F of size $|F| \geq n$ and a set $S = \{a_1, \ldots, a_n\} \subseteq F$ we define the Reed-Solomon code

$$RS_{F,S}[n,k] = \{(P(a_1), \ldots, P(a_n)) \in F^n | P \in F[X] \text{ is a polynomial of degree} \leq k - 1\}$$

A natural interpretation of the $RS_{F,S}[n,k]$ code is via its encoding map. To encode a message $m = \{m_0, \ldots, m_{k-1}\} \in F^k$, we interpret the message as the polynomial

$$P(X) = m_0 + m_1 X + \cdots + m_{k-1} X^{K-1} \in F[X]$$

We then evaluate the polynomial p at the points $a_1, \ldots, a_n$ to get the code word corresponding to m. To evaluate the polynomial p on the points $a_1, \ldots, a_n$, we multiply the message vector m on the left by the $n \times k$ Vandermonde matrix

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$$G = \begin{pmatrix} 1 & a_1 & a_1^2 & a_1^{k-1} \\ 1 & a_2 & a_2^2 & a_2^{k-1} \\ \dots & \dots & \dots & \dots \\ 1 & a_n & a_n^2 & a_n^{k-1} \end{pmatrix}$$

The matrix G is a generator matrix for $RS_{F,S}[n,k]$, so we immediately obtain that Reed-Solomon codes are linear codes over F [7].

Discrete Cosine Transform

With the character of discrete Fourier transform (DFT), discrete cosine transform (DCT) turn over the image edge to make the image transformed into the form of even function. It's one of the most common linear transformations in digital signal process technology. Two dimensional discrete cosine transform (2D-DCT) is defined as

$$F(jk)$$
$$= a(j)a(k) \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} f(mn)cos \frac{(2m+1)j\pi}{2N}$$
$$cos \frac{(2n+1)j\pi}{2N}$$

The corresponding inverse transformation (Whether 2D-IDCT) is defined as

$$f(jk)$$
$$= \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} a(j)a(k)F(jk)cos \frac{(2m+1)j\pi}{2N}$$
$$cos \frac{(2n+1)j\pi}{2N}$$

The 2D-DCT can not only concentrate the main information of original image into the smallest low-frequency coefficient, but also it can cause the image blocking effect being the smallest, which can realize the good compromise between the information centralizing and the computing complication. So it obtains the wide spreading application in the compression coding.

Discrete wavelet transform

Wavelet transform has been widely studied in signal processing in general and image compression in particular. A discrete wavelet transform (DWT) is a sampled wavelet function. Rather than calculate the wavelet coefficients at every point, the DWT uses only a subset of positions and scales. This method results in an accurate and more efficient manner of a wavelet transform. The DWT is similar but more versatile that the Fourier series. The DWT can be made periodic but it can also be applied to non-periodic transient signals.

The DWT of a signal is calculated by passing it through a series of filters. First the samples are passed through a low pass filter with impulse response $'g'$ resulting in a convolution of the two:

$$y[n] = (x * g)[n] = \sum_{k=-\infty}^{\infty} x[n]g[n-k]$$

The signal is also decomposed simultaneously using a high-pass filter $'h'$. The outputs giving the detail coefficients (from the high-pass filter) and approximation coefficients (from the low-pass). It is important that the two filters are related to each other and they are known as a quadrature mirror filter.

However, since half the frequencies of the signal have now been removed, half the samples can be discarded according to Nyquist's rule. The filter outputs are then subsampled by 2 (Mallat's and the common notation is the opposite, g − high pass and h − low pass):

$$y_{low}[n] = \sum_{k=-\infty}^{\infty} x[n]g[2n-k]$$
$$y_{high}[n] = \sum_{k=-\infty}^{\infty} x[n]h[2n-k]$$

This decomposition has halved the time resolution since only half of each filter output characterises the signal.

Here DWT2 (Two dimensional Discrete Wavelet Transform) method is used to decompose the image into four sub bands namely LL, LH, HL & HH.

LL - Low frequency band

LH - Horizontal high frequency band

HL - Vertical high frequency band

HH - Diagonal high frequency band

Wavelet coding schemes are especially suitable for applications where scalability and tolerable degradation are important.
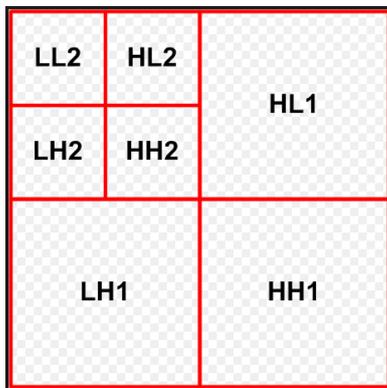
Fig.3. Decomposition using DWT

Characteristics of DWT

1. The wavelet transform decomposes the image into three spatial directions i.e. horizontal, vertical and diagonal. Hence wavelets reflect the anisotropic properties of HVS more precisely.
2. Watermark detection at lower resolutions is computationally effective because at every successive resolution level there are few frequency bands involved.
3. As LL band contains largest wavelet coefficients, scale factor is chosen accordingly up to 0.05 for LL and 0.005 for other bands. For this pair of values, there is no degradation in watermarked image.
4. High resolution sub bands locate edge and textures patterns in an image.
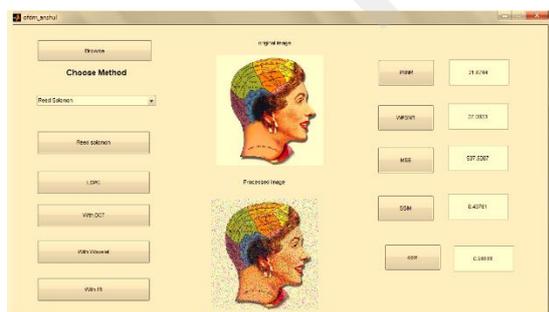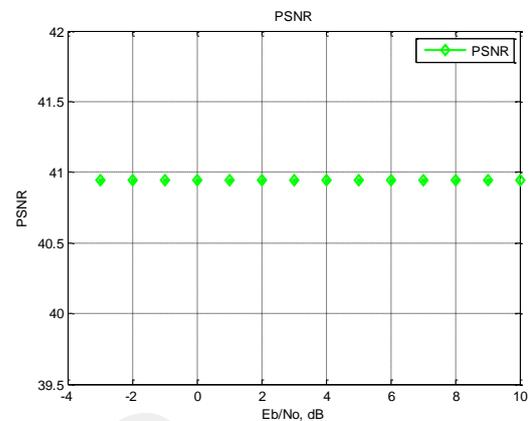

Fig.4.Figure Window in MATLAB
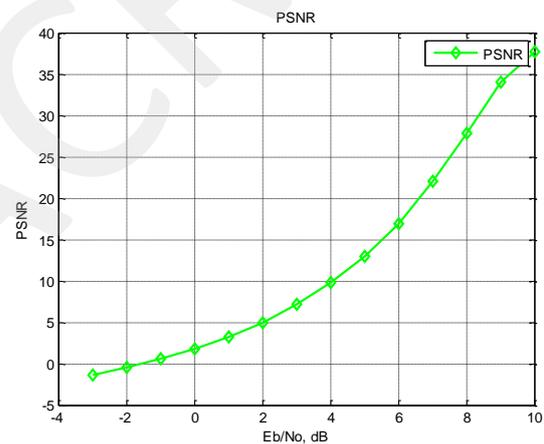
III. SIMULATION RESULTS

Fig.5. PSNR in case of DWT

Fig.6. PSNR in case of DCT
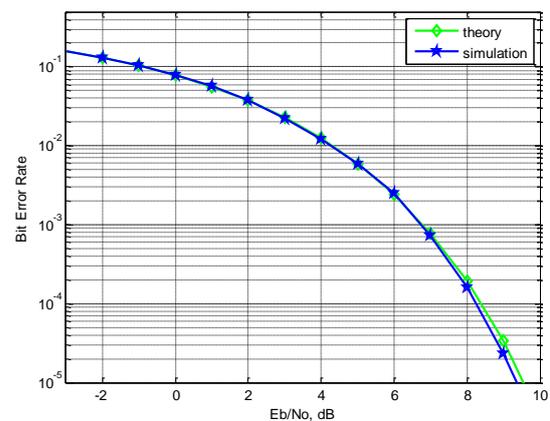

Fig.7. Gaussian Noise BER curve for wavelet source coding with OFDM

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IV. CONCLUSION

We use different error correcting codes; Reed Solomon, LDPC, DCT and DWT. The simulation results shows that the DWT gives better result than the DCT. We found a constant higher PSNR in DWT. It shows the greater efficiency.

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