

Performance Analysis of MC-CDMA by using Rake Receiver

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Abstract – This paper proposes a novel approach for minimizing the effect of multipath fading in Rayleigh fading environment using Multicarrier CDMA (MC-CDMA) with RAKE receiver and different combining schemes. Equal Gain Combining (EGC), Maximal Ratio Combining (MRC), Zero-Forcing (Z-F) and Minimum Mean Square Error (MMSE) Equalization techniques are used.

Keywords – MC-CDMA, RAKE Receiver, EGC, MRC, Z-F, MMSE.

I. INTRODUCTION

Multicarrier code-division multiple access (MCCDMA) is a promising approach to the challenge of providing high data rate wireless communication. MC-CDMA combines the benefits of CDMA with the natural robustness to frequency selectivity offered by OFDM. It can be interpreted as CDMA with the spreading taking place in the frequency rather than temporal domain. Multicarrier CDMA is fusion of two different techniques:

1. Orthogonal Frequency Division Multiplexing (OFDM)
2. Code Division Multiple Access (CDMA)

Multi-carrier CDMA is a digital modulation technique where a single data symbol is transmitted at multiple narrowband subcarriers with each subcarrier encoded with a phase offset of 0 or π based on a spreading code. The narrowband subcarriers are generated using BPSK modulated signals, each at different frequencies which at baseband are at multiples of a harmonic frequency. Consequently, the subcarriers are orthogonal to each other at baseband, and the component at each subcarrier may be filtered out by modulating the received signal with the frequency corresponding to the particular subcarrier of interest and integrating over a symbol duration. The orthogonality between subcarrier frequencies is maintained if the subcarrier frequencies are spaced apart by multiples of F/T_b where F is an integer.

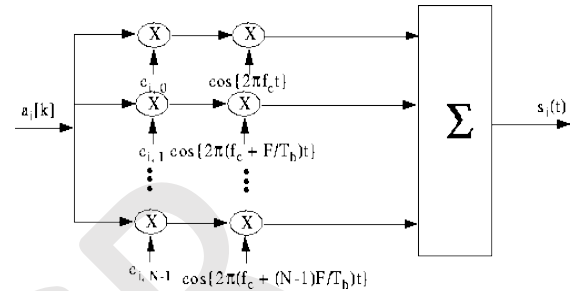


Figure 1: MC-CDMA Transmitter Model

Fig1 shows the transmitter model for one possible implementation of an MC-CDMA system. The input data symbols, $a_i[k]$, are assumed to be binary where k denotes the k^{th} bit interval and i denotes the i^{th} user. In the analysis, it is assumed that $a_i[k]$ takes on values of -1 and 1 with equal probability.

In MC-CDMA system a single data symbol is replicated into N parallel copies. The i th branch (subcarrier) of the parallel stream is multiplied by a chip, $c_i[m]$ from a pseudo-random (PN) code or some other orthogonal code of length N and then BPSK modulated to a subcarrier spaced apart from its neighboring subcarriers by F/T_b where F is an integer number. The transmitted signal consists of the sum of the outputs of these branches. This process yields a multicarrier signal with the subcarriers containing the PN-coded data symbol.

The transmitted signal is given by:

$$S_i(t) = + \sum_{m=1}^{N-1} \left(c_i[m] a_i[k] \cos \left(2\pi f_c t + 2\pi m \frac{F}{T_b} t \right) p_{T_b}(t - kT_b) \right) \quad [1]$$

$c_i[m] \in \{-1, 1\}$

Where $c_i[0], c_i[1], \dots, c_i[N-1]$ represents the spreading code of the i th user and $p_{T_b}(t)$ is defined to be an unit amplitude pulse that is non-zero in the interval of $[0, T_b]$.

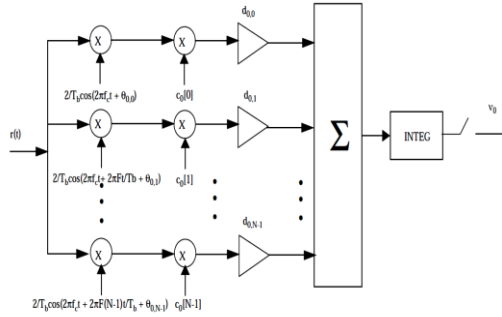


Figure 2: Receiver Model

When there are M active users, the received signal is

$$r(t) = \sum_{m=0}^{M-1} \sum_{i=0}^{N-1} \rho_{m,i} c_m[i] a_m[k] \cos \left(2\pi f_c t + 2\pi m \frac{F}{T_b} t + \theta_{m,i} \right) + n(t) \quad [2]$$

Where the effects of the channel have been included in $\rho_{m,i}$ and $\theta_{m,i}$ and $n(t)$ is additive white Gaussian noise (AGWN) with a one-sided power spectral density of N_0 .

Assuming the transmitter model of Fig.1, a possible implementation of the receiver is shown in Fig.2 where it has been assumed that $m = 0$ corresponds to the desired signal. With this model, there are N matched filters with one matched filter for each subcarrier. The output of each filter contributes one component to the decision variable v_0 . Each matched filter consists of an oscillator with a frequency corresponding to the frequency of the particular BPSK modulated subcarrier that is of interest and an integrator. In addition, a phase offset equal to the phase distortion introduced by the channel, $\theta_{0,i}$, is included in the oscillator to synchronize the receiver to the desired signal in time. To extract the desired signal's component, the orthogonality of the codes is used. For the i th subcarrier of the desired signal, the corresponding chip, $c_0[i]$, from the desired user's code is multiplied with it to undo the code. If the signal is undistorted by the channel, the interference terms will cancel out in the decision variable due to the orthogonality of the codes.

As the channel will distort the subcarrier components, an equalization gain, $d_{0,i}$, may be included for each matched filter branch of the receiver.

Applying the receiver model of Fig.2 to the received signal given in Eq.[2] yields the following decision variable for the k^{th} data symbol assuming the users are synchronized in time

$$v_0 = \sum_{m=0}^{M-1} \sum_{i=0}^{N-1} \rho_{m,i} c_m[i] d_{0,i} a_m[k] \frac{2}{T_b} \int_{kT_b}^{(k+1)T_b} \cos \left(2\pi f_c t + 2\pi m \frac{F}{T_b} t + \theta_{m,i} \right) dt + \eta \quad [3]$$

Where $\hat{\theta}_{0,i}$ denotes the receiver's estimation of the phase at the i th subcarrier of the desired signal and the corresponding AWGN term, η , is given as:

$$\eta = \sum_{i=0}^{N-1} \int_{kT_b}^{(k+1)T_b} n(t) \frac{2}{T_b} d_{0,i} \cos \left(2\pi f_c t + 2\pi m \frac{F}{T_b} t + \hat{\theta}_{0,i} \right) dt \quad [4]$$

II. RAKE RECEIVER

The rake receiver consists of multiple correlators, in which the receive signal is multiplied by time-shifted versions of a locally generated code sequence. The intention is to separate signals such that each finger only sees signals coming in over a single (resolvable) path. The spreading code is chosen to have a very small autocorrelation value for any nonzero time offset. This avoids crosstalk between fingers. In practice, the situation is less ideal. It is not the full periodic autocorrelation that determines the crosstalk between signals in different fingers, but rather two *partial* correlations, with contributions from two consecutive bits or symbols. It has been attempted to find sequences that have satisfactory partial correlation values, but the crosstalk due to partial (non-periodic) correlations remains substantially more difficult to reduce than the effects of periodic correlations.

The rake receiver is designed to optimally detect a DS-CDMA signal transmitted over a dispersive multipath channel. It is an extension of the concept of the matched filter.

In the matched filter receiver, the signal is correlated with a locally generated copy of the signal waveform. If, however, the signal is distorted by the channel, the receiver should correlate the incoming signal by a copy of the expected received signal, rather than by a copy of transmitted waveform. Thus the receiver should estimate the delay profile of channel, and adapt its locally generated copy according to this estimate.

In a multipath channel, delayed reflections interfere with the direct signal. However, a DS-CDMA signal

suffering from multipath dispersion can be detected by a rake receiver. This receiver optimally combines signals received over multiple paths.

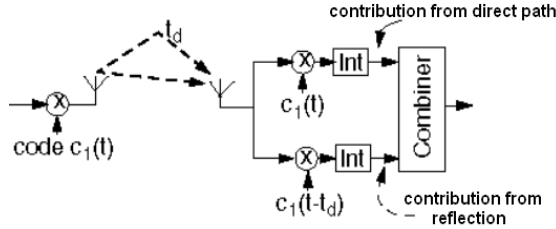


Figure 3: RAKE Receiver

Rake receiver gathers the energy received over the various delayed propagation paths. According to the maximum ratio combining principle, the SNR at the output is the sum of the SNRs in the individual branches, provided that

1. Only AWGN is present (no interference)
2. Codes with a time offset are truly orthogonal.

Signals arriving with the same excess propagation delay as the time offset in the receiver are retrieved accurately, because

$$\sum_{n=1}^N c_1^2(nT_c + t_d) = \sum_{n=1}^N c_1^2(nT_c) = N \quad [5]$$

This reception concept is repeated for every delayed path that is received with relevant power. Considering a single correlator branch, multipath self-interference from other paths is attenuated here, because one can choose codes such that:

$$\sum_{n=0}^N c_1(nT_c) c_1(nT_c + t_d) \cong 0 \quad [6]$$

III. EQUALIZATION TECHNIQUES

The underlying goal of equalization techniques should be to reduce the effect of the fading and the interference while not enhancing the effect of the noise on the decision of what data symbol was transmitted. Whenever there is a diversity scheme involved whether it may involve receiving multiple copies of a signal from time, frequency or antenna diversity, the field of classical diversity theory can be applied. These equalization techniques may be desirable for their simplicity as they involve simple multiplications with each copy of the signal. However, they may not be optimal in a channel with

interference in the sense of minimizing the error under some criterion. In some ways, these equalization schemes appear "ad hoc" as they are not derived under some clear procedure.

Equal Gain Combining

With EGC, the gain factor of the i th subcarrier is chosen to be

$$d_{o,i} = 1$$

That is, this technique does not attempt to equalize the effect of the channel distortion in any way. This technique may be desirable for its simplicity as the receiver does not require the estimation of the channel's transfer function. Using this scheme, the decision variable of Eq. (3) is given as:

$$\begin{aligned} v_0 &= a_0[k] \sum_{i=0}^{N-1} \rho_{0,i} \\ &+ \sum_{m=0}^{M-1} a_m[k] \sum_{i=0}^{N-1} c_m[i] c_0[i] \rho_{m,i} \cos \hat{\theta}_{m,i} + \eta \end{aligned} \quad [7]$$

Where the noise can be approximated by a zero mean Gaussian random with a variance of

$$\sigma_\eta^2 = N \frac{N_0}{T_b}$$

Maximal Ratio Combining

With MRC, the scheme squares the amplitude of each copy of the signal by using a gain factor for the i th subcarrier of

$$d_{o,i} = \rho_{0,i}$$

The motivation behind Maximal Ratio Combining is that the components of the received signal with large amplitudes are likely to contain relatively less noise. Thus, their effect on the decision process is increased by squaring their amplitudes. The corresponding decision variable is:

$$\begin{aligned} v_0 &= a_0[k] \sum_{i=0}^{N-1} \rho_{0,i} \\ &+ \sum_{m=0}^{M-1} a_m[k] \sum_{i=0}^{N-1} c_m[i] c_0[i] \rho_{m,i} \rho_{0,i} \cos \hat{\theta}_{m,i} + \eta \end{aligned} \quad [8]$$

Where the noise can be approximated by a zero-mean Gaussian random variable with variance

$$\sigma_\eta^2 = N \frac{N_0}{T_b} E \rho_{0,i}^2 \quad [9]$$

Zero-Forcing

Zero-Forcing (ZF) technique is the simplest MIMO detection technique, Where filtering matrix is constructed using the ZF performance based criterion. The drawback of ZF scheme is the susceptible noise enhancement and loss of diversity order due to linear filtering [4], [5]. ZF can be implemented by using the inverse of the channel matrix H to produce the estimate of transmitted vector $\tilde{x}$

$$\begin{aligned}\tilde{x} &= H^\dagger r \\ &= H^\dagger (Hx) \\ &= x\end{aligned}$$

Where $(.)^\dagger$ denotes the pseudo-inverse. But when the noise term is considered, the post-processing signal is given by:

$$\begin{aligned}\tilde{x} &= H^\dagger R \\ &= H^\dagger (Hx+n) \\ &= x + H^\dagger n\end{aligned}$$

With the addition of the noise vector, ZF estimate, i.e. $\tilde{x}$ consists of the decoded vector x plus a combination of the inverted channel matrix and the unknown noise vector. Because the pseudo-inverse of the channel matrix may have high power when the channel matrix is ill-conditioned, the noise variance is consequently increased and the performance is degraded. To alleviate for the noise enhancement introduced by the ZF detector, the MMSE detector was proposed, where the noise variance is considered in the construction of the filtering matrix G .

Minimum Mean Square Error

Minimum Mean Square Error (MMSE) approach alleviates the noise enhancement problem by taking into consideration the noise power when constructing the filtering matrix using the MMSE performance-based criterion. The vector estimates produced by an MMSE filtering matrix becomes

$$\tilde{x} = [(H^H H + (\sigma^2 I))^{-1}] H^H r \quad [10]$$

Where σ^2 is the noise variance. The added term $(1/SNR = \sigma^2, \text{ in case of unit transmit power})$ offers a trade-off between the residual interference and the noise enhancement. Namely, as the SNR grows large, the MMSE detector converges to the ZF detector, but at low SNR it prevents the worst Eigen

values from being inverted. At low SNR, MMSE becomes Matched Filter

$$[(H^H H + (\sigma^2 I))^{-1}] H^H \approx \sigma^2 H^H \quad [11]$$

At high SNR, MMSE becomes ZF:

$$(H^H H + (\sigma^2 I))^{-1} H^H \approx (H^H H)^{-1} H^H \quad [12]$$

IV. SIMULATION AND RESULTS

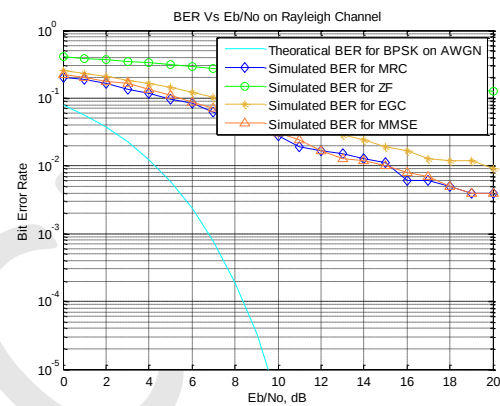


Figure 4: BER analysis for 4 finger

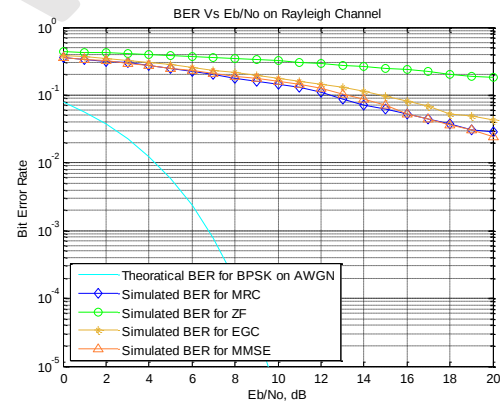


Figure 5: BER analysis for 2 finger

V. CONCLUSION

This paper concludes the performance evaluation of MC-CDMA with Rake receiver concept for various equalization techniques. As we increase the no. of finger, BER performance improves. Rake receiver with MMSE gives better BER performance as we increase SNR.

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